

# 11.2 Areas of Trapezoids, Rhombuses, and Kites



**Before**

You found areas of triangles and parallelograms.

**Now**

You will find areas of other types of quadrilaterals.

**Why?**

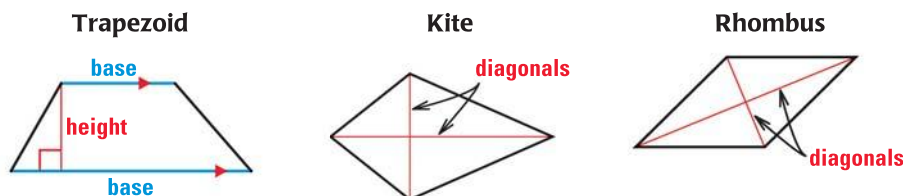
So you can solve problems that involve area, as in Example 1.

## Key Vocabulary

- **height of a trapezoid**
- **diagonal**, p. 507
- **bases of a trapezoid**, p. 542

As you saw in the Activity on page 729, you can use the area formula for a parallelogram to develop area formulas for other special quadrilaterals. The areas of the figures below are related to the lengths of the marked segments.

The **height of a trapezoid** is the perpendicular distance between its bases.



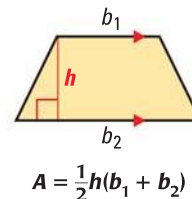
## THEOREM

## For Your Notebook

### THEOREM 11.4 Area of a Trapezoid

The area of a trapezoid is one half the product of the height and the sum of the lengths of the bases.

*Proof:* Ex. 40, p. 736



## EXAMPLE 1 Find the area of a trapezoid

**BASKETBALL** The free-throw lane on an international basketball court is shaped like a trapezoid. Find the area of the free-throw lane.

### Solution

The height of the trapezoid is 5.8 meters. The lengths of the bases are 3.6 meters and 6 meters.

$$A = \frac{1}{2}h(b_1 + b_2)$$

**Formula for area of a trapezoid**

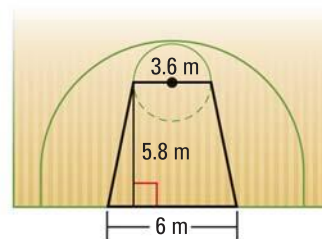
$$= \frac{1}{2}(5.8)(3.6 + 6)$$

**Substitute 5.8 for  $h$ , 3.6 for  $b_1$ , and 6 for  $b_2$ .**

$$= 27.84$$

**Simplify.**

► The area of the free-throw lane is about 27.8 square meters.



## ANOTHER WAY

In a trapezoid, the average of the lengths of the bases is also the length of the midsegment. So, you can also find the area by multiplying the midsegment by the height.

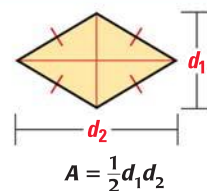
**ANOTHER WAY**

Remember that a rhombus is also a parallelogram, so you can also use the formula  $A = bh$ .

**THEOREMS***For Your Notebook***THEOREM 11.5 Area of a Rhombus**

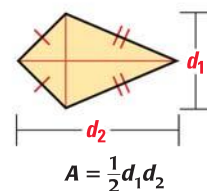
The area of a rhombus is one half the product of the lengths of its diagonals.

*Justification:* Ex. 39, p. 735

**THEOREM 11.6 Area of a Kite**

The area of a kite is one half the product of the lengths of its diagonals.

*Proof:* Ex. 41, p. 736

**EXAMPLE 2 Find the area of a rhombus**

**MUSIC** Rhombus  $PQRS$  represents one of the inlays on the guitar in the photo. Find the area of the inlay.

**Solution**

**STEP 1** Find the length of each diagonal. The diagonals of a rhombus bisect each other, so  $QN = NS$  and  $PN = NR$ .

$$QS = QN + NS = 9 + 9 = 18 \text{ mm}$$

$$PR = PN + NR = 12 + 12 = 24 \text{ mm}$$

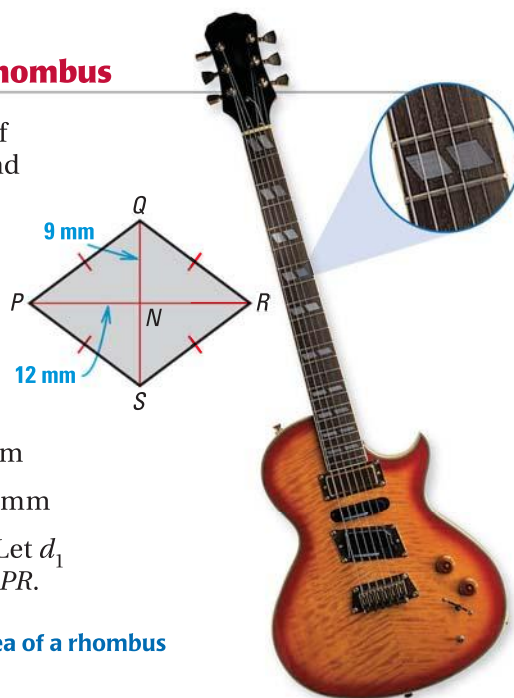
**STEP 2** Find the area of the rhombus. Let  $d_1$  represent  $QS$  and  $d_2$  represent  $PR$ .

$$A = \frac{1}{2}d_1d_2 \quad \text{Formula for area of a rhombus}$$

$$= \frac{1}{2}(18)(24) \quad \text{Substitute.}$$

$$= 216 \quad \text{Simplify.}$$

► The area of the inlay is 216 square millimeters.

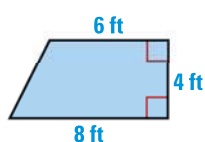
**READ DIAGRAMS**

When you read a diagram, look for information you need to find. The diagram gives the lengths of  $\overline{QN}$  and  $\overline{PN}$ , but not the lengths of  $\overline{QS}$  and  $\overline{PR}$ .

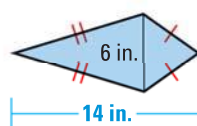
**GUIDED PRACTICE for Examples 1 and 2**

Find the area of the figure.

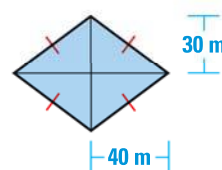
1.



2.



3.



**EXAMPLE 3** Standardized Test Practice**ELIMINATE CHOICES**

In Example 3, you can eliminate choices A and B because in each case, one diagonal is not twice as long as the other diagonal.

One diagonal of a kite is twice as long as the other diagonal. The area of the kite is 72.25 square inches. What are the lengths of the diagonals?

- (A) 6 in., 6 in. (B) 8.5 in., 8.5 in. (C) 8.5 in., 17 in. (D) 6 in., 12 in.

**Solution**

Draw and label a diagram. Let  $x$  be the length of one diagonal. The other diagonal is twice as long, so label it  $2x$ . Use the formula for the area of a kite to find the value of  $x$ .

$$A = \frac{1}{2}d_1d_2 \quad \text{Formula for area of a kite}$$

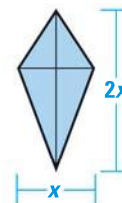
$$72.25 = \frac{1}{2}(x)(2x) \quad \text{Substitute 72.25 for } A, x \text{ for } d_1, \text{ and } 2x \text{ for } d_2.$$

$$72.25 = x^2 \quad \text{Simplify.}$$

$$8.5 = x \quad \text{Find the positive square root of each side.}$$

The lengths of the diagonals are 8.5 inches and  $2(8.5) = 17$  inches.

► The correct answer is C. (A) (B) (C) (D)

**EXAMPLE 4** Find an area in the coordinate plane

**CITY PLANNING** You have a map of a city park. Each grid square represents a 10 meter by 10 meter square. Find the area of the park.

**Solution**

**STEP 1** Find the lengths of the bases and the height of trapezoid  $ABCD$ .

$$b_1 = BC = |70 - 30| = 40 \text{ m}$$

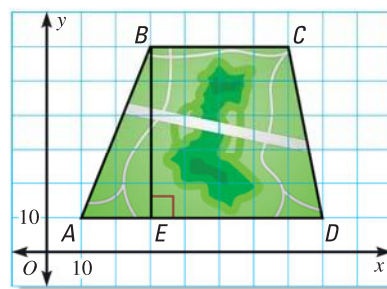
$$b_2 = AD = |80 - 10| = 70 \text{ m}$$

$$h = BE = |60 - 10| = 50 \text{ m}$$

**STEP 2** Find the area of  $ABCD$ .

$$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(50)(40 + 70) = 2750$$

► The area of the park is 2750 square meters.

**GUIDED PRACTICE** for Examples 3 and 4

- The area of a kite is 80 square feet. One diagonal is 4 times as long as the other. Find the diagonal lengths.
- Find the area of a rhombus with vertices  $M(1, 3)$ ,  $N(5, 5)$ ,  $P(9, 3)$ , and  $Q(5, 1)$ .

# 11.2 EXERCISES

## HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS**  
on p. WS1 for Exs. 9, 17, and 35

★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 15, 30, 39, and 42

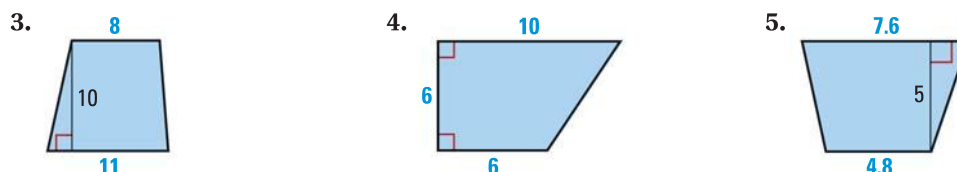
## SKILL PRACTICE

### EXAMPLE 1

on p. 730  
for Exs. 3–6

- VOCABULARY** Copy and complete: The perpendicular distance between the bases of a trapezoid is called the   ?   of the trapezoid.
- ★ **WRITING** Sketch a kite and its diagonals. Describe what you know about the segments and angles formed by the intersecting diagonals.

### FINDING AREA Find the area of the trapezoid.

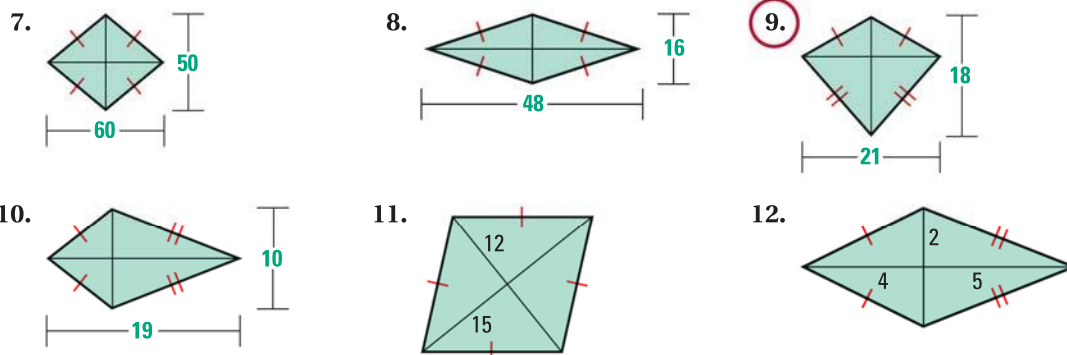


- DRAWING DIAGRAMS** The lengths of the bases of a trapezoid are 5.4 centimeters and 10.2 centimeters. The height is 8 centimeters. Draw and label a trapezoid that matches this description. Then find its area.

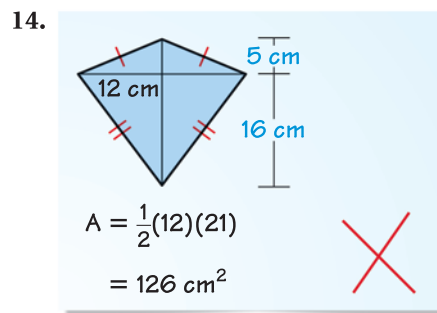
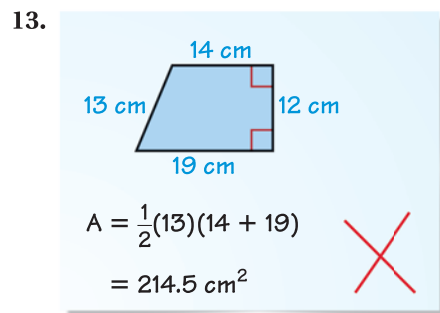
### EXAMPLE 2

on p. 731  
for Exs. 7–14

### FINDING AREA Find the area of the rhombus or kite.



### ERROR ANALYSIS Describe and correct the error in finding the area.



### EXAMPLE 3

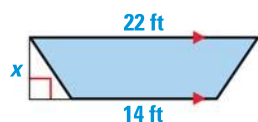
on p. 732  
for Exs. 15–18

- ★ **MULTIPLE CHOICE** One diagonal of a rhombus is three times as long as the other diagonal. The area of the rhombus is 24 square feet. What are the lengths of the diagonals?
 

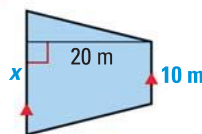
(A) 8 ft, 11 ft      (B) 4 ft, 12 ft      (C) 2 ft, 6 ft      (D) 6 ft, 24 ft

**xy ALGEBRA** Use the given information to find the value of  $x$ .

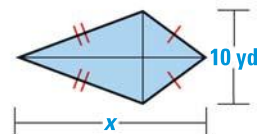
16. Area =  $108 \text{ ft}^2$



17. Area =  $300 \text{ m}^2$



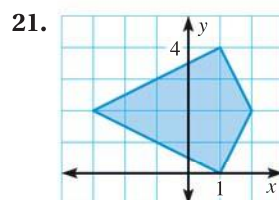
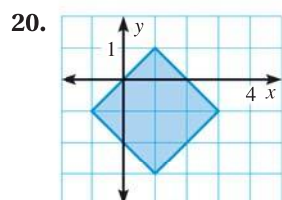
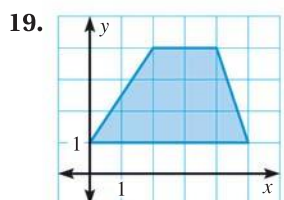
18. Area =  $100 \text{ yd}^2$



**EXAMPLE 4**

on p. 732  
for Exs. 19–21

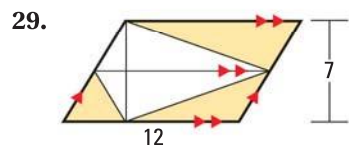
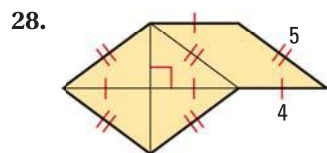
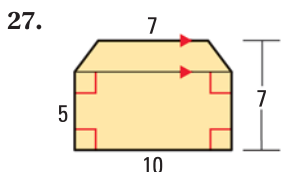
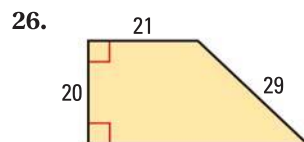
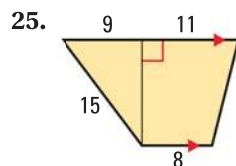
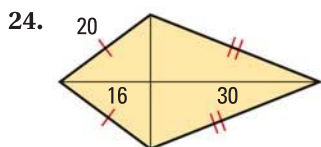
**COORDINATE GEOMETRY** Find the area of the figure.



**xy ALGEBRA** Find the lengths of the bases of the trapezoid described.

22. The height is 3 feet. One base is twice as long as the other base. The area is 13.5 square feet.
23. One base is 8 centimeters longer than the other base. The height is 6 centimeters and the area is 54 square centimeters.

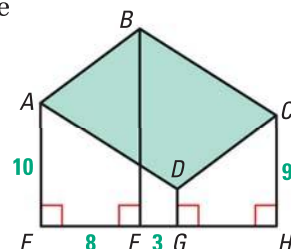
**FINDING AREA** Find the area of the shaded region.



30. **★ OPEN-ENDED MATH** Draw three examples of trapezoids that match this description: The height of the trapezoid is 3 units and its area is the same as the area of a parallelogram with height 3 units and base 8 units.

**VISUALIZING** Sketch the figure. Then determine its perimeter and area.

31. The figure is a trapezoid. It has two right angles. The lengths of its bases are 7 and 15. Its height is 6.
32. The figure is a rhombus. Its side length is 13. The length of one of its diagonals is 24.
33. **CHALLENGE** In the diagram shown at the right,  $ABCD$  is a parallelogram and  $BF = 16$ . Find the area of  $\square ABCD$ . Explain your reasoning. (Hint: Draw auxiliary lines through point  $A$  and through point  $D$  that are parallel to  $\overline{EH}$ .)





## PROBLEM SOLVING

### EXAMPLE 1

on p. 730  
for Ex. 34

- 34. TRUCKS** The windshield in a truck is in the shape of a trapezoid. The lengths of the bases of the trapezoid are 70 inches and 79 inches. The height is 35 inches. Find the area of the glass in the windshield.

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### EXAMPLE 2

on p. 731  
for Ex. 35

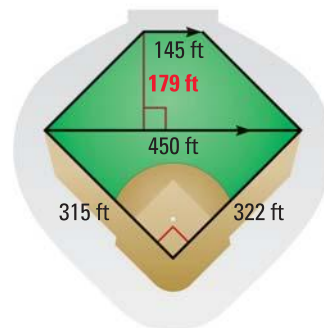
- 35. INTERNET** You are creating a kite-shaped logo for your school's website. The diagonals of the logo are 8 millimeters and 5 millimeters long. Find the area of the logo. Draw two different possible shapes for the logo.

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- 36. DESIGN** You are designing a wall hanging that is in the shape of a rhombus. The area of the wall hanging is 432 square inches and the length of one diagonal is 36 inches. Find the length of the other diagonal.

- 37. MULTI-STEP PROBLEM** As shown, a baseball stadium's playing field is shaped like a pentagon. To find the area of the playing field shown at the right, you can divide the field into two smaller polygons.

- Classify the two polygons.
- Find the area of the playing field in square feet. Then express your answer in square yards. Round to the nearest square foot.



- 38. VISUAL REASONING** Follow the steps in parts (a)–(c).

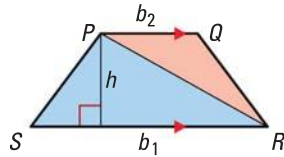
- Analyze** Copy the table and extend it to include a column for  $n = 5$ . Complete the table for  $n = 4$  and  $n = 5$ .

Rhombus number, $n$	1	2	3	4
Diagram				
Area, $A$	2	4	6	?

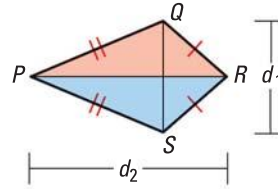
- Use Algebra** Describe the relationship between the rhombus number  $n$  and the area of the rhombus. Then write an algebraic rule for finding the area of the  $n$ th rhombus.
  - Compare** In each rhombus, the length of one diagonal ( $d_1$ ) is 2. What is the length of the other diagonal ( $d_2$ ) for the  $n$ th rhombus? Use the formula for the area of a rhombus to write a rule for finding the area of the  $n$ th rhombus. Compare this rule with the one you wrote in part (b).
- 39. ★ SHORT RESPONSE** Look back at the Activity on page 729. Explain how the results for kites in Explore 2 can be used to justify Theorem 11.5, the formula for the area of a rhombus.

**PROVING THEOREMS 11.4 AND 11.6** Use the triangle area formula and the triangles in the diagram to write a plan for the proof.

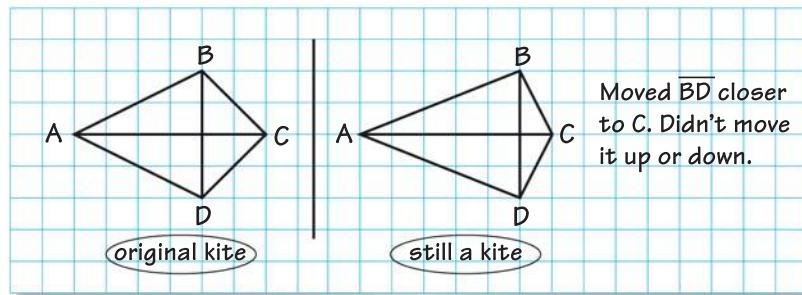
40. Show that the area  $A$  of the trapezoid shown is  $\frac{1}{2}h(b_1 + b_2)$ .



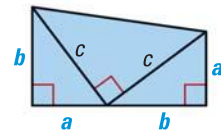
41. Show that the area  $A$  of the kite shown is  $\frac{1}{2}d_1d_2$ .



42. ★ **EXTENDED RESPONSE** You will explore the effect of moving a diagonal.



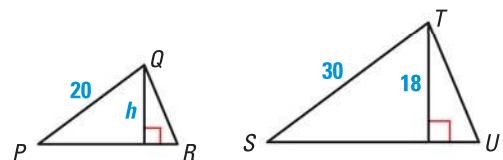
- a. **Investigate** Draw a kite in which the longer diagonal is horizontal. Suppose this diagonal is fixed and you can slide the vertical diagonal left or right and up or down. You can keep sliding as long as the diagonals continue to intersect. Draw and identify each type of figure you can form.
- b. **Justify** Is it possible to form any shapes that are not quadrilaterals? *Explain.*
- c. **Compare** Compare the areas of the different shapes you found in part (b). What do you notice about the areas? *Explain.*
43. **CHALLENGE** James A. Garfield, the twentieth president of the United States, discovered a proof of the Pythagorean Theorem in 1876. His proof involved the fact that a trapezoid can be formed from two congruent right triangles and an isosceles right triangle. Use the diagram to show that  $a^2 + b^2 = c^2$ .



## MIXED REVIEW

Solve for the indicated variable. Write a reason for each step. (p. 105)

44.  $d = rt$ ; solve for  $t$       45.  $A = \frac{1}{2}bh$ ; solve for  $h$       46.  $P = 2\ell + 2w$ ; solve for  $w$
47. Find the angle measures of an isosceles triangle if the measure of a base angle is 4 times the measure of the vertex angle. (p. 264)
48. In the diagram at the right,  $\triangle PQR \sim \triangle STU$ . The perimeter of  $\triangle STU$  is 81 inches. Find the height  $h$  and the perimeter of  $\triangle PQR$ . (p. 372)



### PREVIEW

Prepare for  
Lesson 11.3 in  
Ex. 48.

